Mathematical Modeling of Social Phenomena

Evolutionary Game Theory

Lecture layout

Game theory repetition
ESS
Replicator dynamics
Discussing your final papers

What is a game?

A set of players:

$P = \{p_1, \ldots, p_k\}$

A set of strategies:

$S = \{s_1, \ldots, s_n\}$

A mapping from a tuple of selected strategies to a payoff:

$\square : S \times \ldots \times S \rightarrow \mathbf{R}$

Payoff versus utility

The payoff function:

$\pi : S \times \ldots \times S \rightarrow \mathbf{R}$

But what is the utility function?

Altruism?

Morality?

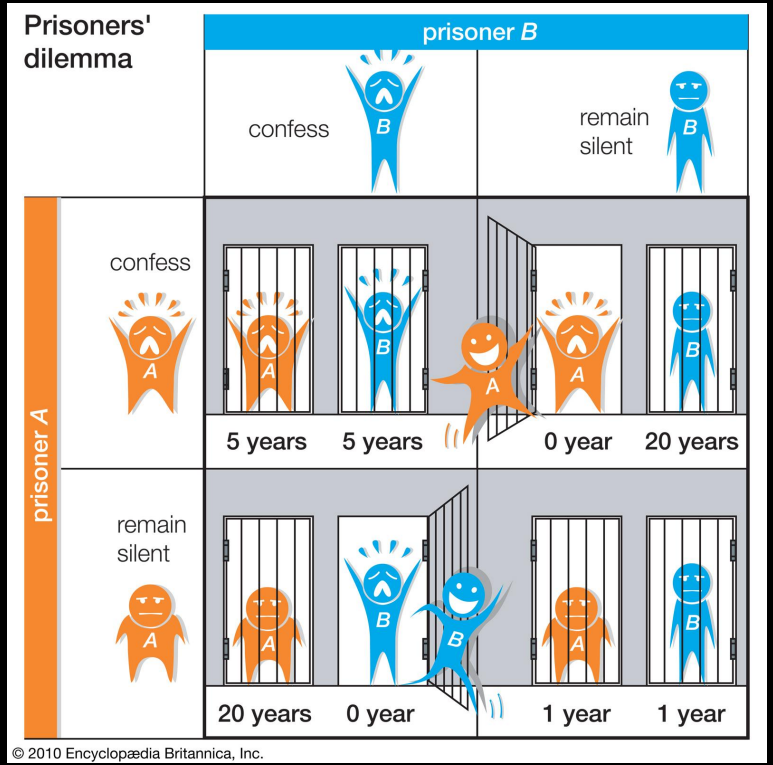
How does the book chapter handle this?

On dominant strategies

A strategy tuple (s_1^*, s_2^*) in S x S is a Nash equilibrium if for all players i and strategies s_i,

$$\pi_i(s_1^*, s_2^*) \geq \pi_i(s_i, s_2^*)$$

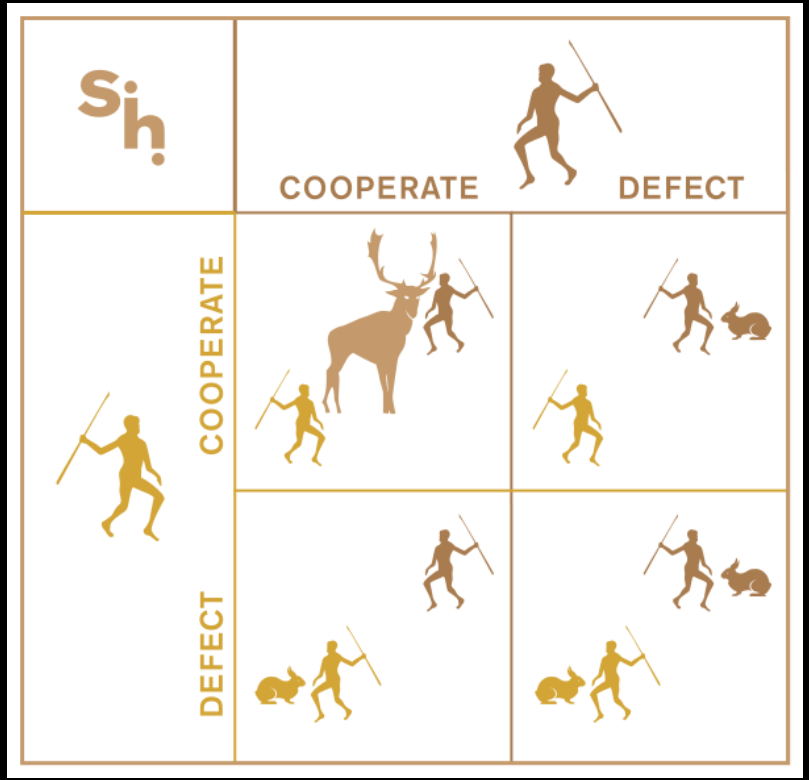
I.e., no one has an incentive to independently change their strategy.



Risk- v. payoff dominant

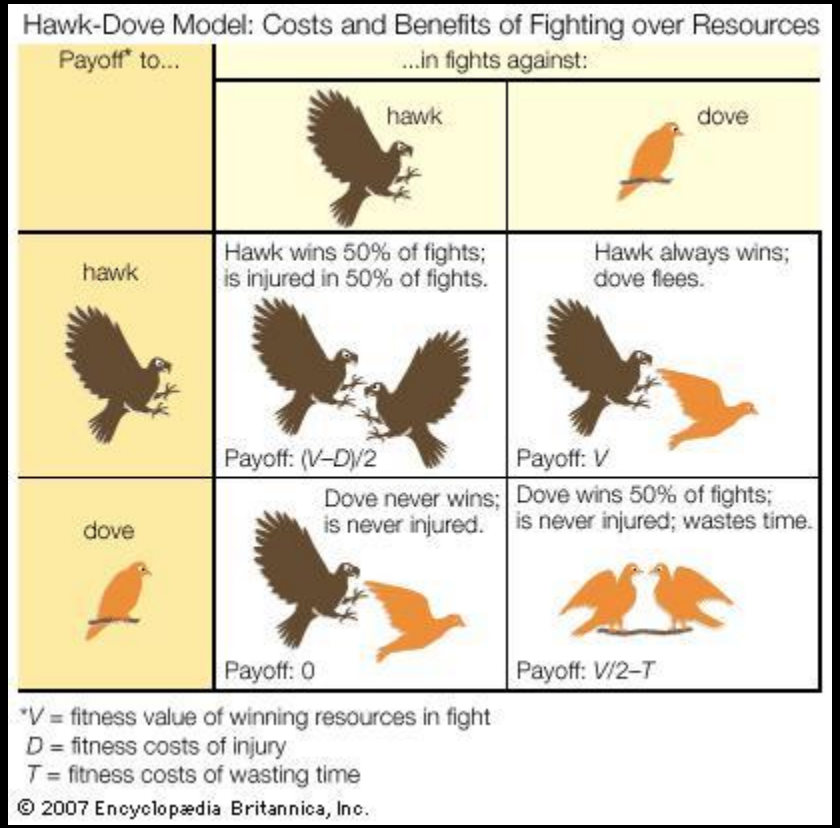
Harder to coordinate, because we don't know if the others are either or?

Risk dominance tend to win out.



Anti-coordination game

Pre-commitment



Mixed strategy

2 0

0 1

What are the three equilibria?

Optimality

Individual

Pareto

A choice of strategies—one by each player—is Pareto-optimal if there is no other choice of strategies in which all players receive payos at least as high, and at least one player receives a strictly higher pay.

Social

Evolutionary game theory

Or rather, ESS

Fitness as a result of interaction

Beatles size

Game:

5,5	1,8
8,1	3,3

s_1 is an ESS if there is a positive number y such that if any other strategy invades at a level $x < y$, the payoff of s_1 is greater than of the invader

Evolutionary arms races

What are they?

A sign of an intelligent designer?

The ESS conditions

Game: a, a b, c

c, b d, d

In a two-player, two-strategy, symmetric game, s_1 is evolutionary stable precisely when either

$a > c$

or

$a = c$, and $b > d$.

Nash equilibria and ESS

Why is an ESS a Nash equilibria?

In game:

	1, 1	0, 0
	0, 0	2, 2

The three equilibria, are they Nash, are they ESS?

ESSs in the Hawk and dove game

	Dove	Hawk
Dove:	3, 3	1, 5
Hawk	5, 1	0, 0

$$V(p, q) = 3pq + p(1-q) + 5(1-p)q + 0(1-q)(1-p)$$

ExPayoff of Dove: $3p + 1-p = 1+2p$

ExPayoff of Hawk: $5p$.

Implies $p = \frac{1}{3}$.

$V(p, p) = V(q, p)$ for all q, due to the indifference

$\Rightarrow V(p, q) > V(q, q)$?

Actually just go through it as it stands on page 224.

Replicator dynamics